

① Geometric series with first term a and common ratio $r = 3/4$.
Sum of first four terms is 175.

② $a \frac{(1 - (3/4)^4)}{1 - 3/4} = 175 \Leftrightarrow a \left(\frac{175}{64} \right) = 175 \Leftrightarrow a = 64$ [2]

③ $S_{\infty} = \frac{a}{1-r} = \frac{64}{1-3/4} = 256$. [2]

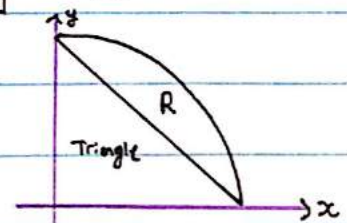
④ Let $u_n = 64 \cdot (3/4)^{n-1}$ then $|u_9 - u_{10}| = 64 |(3/4)^8 - (3/4)^9| = 1.602$ [3]

②

①	x	0	1	2	3	4	where $y = 8 - 2^{x-1}$	[1]
	y	7.5	7	6	4	0		

③ $\int_0^4 8 - 2^{x-1} dx \approx \frac{1}{2} \cdot \frac{(4-0)}{4} [7.5 + 2(7+6+4) + 0] = 20.75$ [3]

④ $R \approx 20.75 - \frac{1}{2} \times 4 \times 7.5 = 5.75$



[2]

$$\textcircled{3} \textcircled{a} |PA| = \sqrt{(10-7)^2 + (13-8)^2} = \sqrt{9 + 25} = \sqrt{34} \quad [2]$$

where $P(7, 8)$ and $A(10, 13)$ and $|PA|$ is the exact length of PA .

$$\textcircled{b} C \text{ is a circle centre } P \text{ and passes through } A \text{ so has radius } r = \sqrt{34}. \\ \therefore C: (x-7)^2 + (y-8)^2 = 34. \quad [2]$$

$$\textcircled{c} \text{ Gradient of radius is } m_r = \frac{13-8}{10-7} = \frac{5}{3}. \quad [4]$$

So gradient of tangent is $m_T = -\frac{3}{5}$ (perpendicular).

$$l \text{ passes through } A \text{ so: } l: y-13 = -\frac{3}{5}(x-10)$$

[Note:
 $\pm k(5y+3x-95)=0$
 $(k=0)$ are valid answers.

$$\Leftrightarrow 5y - 65 = -3x + 30 \Leftrightarrow 5y + 3x - 95 = 0. \quad k \in \mathbb{Z}.]$$

$$\textcircled{4} f(x) = 6x^3 + 13x^2 - 4$$

$$\textcircled{a} \text{ When } f(x) \text{ is divided by } (2x+3) \text{ the remainder is } f(-3/2) = 5. \quad [2]$$

$$\textcircled{b} (x+2) \text{ is a factor of } f(x) \text{ iff } f(-2) = 0. \text{ since } f(-2) = 6(-2)^3 + 13(-2)^2 - 4 \\ = -48 + 52 - 4 = 0 \text{ then } (x+2) \text{ is intd a factor of } f(x) \quad [2]$$

$$\textcircled{c} \text{ Note that: } \begin{array}{r} 6x^2 + x - 2 \\ x+2 \overline{) 6x^3 + 13x^2 + 0x - 4} \\ \underline{6x^3 + 12x^2} \\ x^2 + 0x - 4 \\ \underline{x^2 + 2x} \\ -2x - 4 \\ \underline{-2x - 4} \\ 0 \end{array} \quad \text{So } f(x) = (x+2)(6x^2 + x - 2) \\ = (x+2)(3x+2)(2x-1) // \quad [4]$$

5) (a) first three terms of $(2-9x)^4 = 2^4(1 - \frac{9}{2}x)^4$ [4]

is $2^4 \left[1 - 18x + \frac{243}{2}x^2 + \dots \right] = 2^4 - 288x + 1944x^2$
 $= 16 - 288x + 1944x^2$

$f(x) = (1+Kx)(2-9x)^4 = A - 232x + Bx^2 + \dots$

(b) $A = 2^4 = 16$ [1]

(c) $f(x) = (1+Kx)(16 - 288x + 1944x^2 + \dots) = 16 - 288x + 1944x^2 + 16Kx - 288Kx^2 + \dots$ [2]

so $f(x) = 16 + x(16K - 288) + x^2(1944 - 288K) + \dots$

Comparing coefficients of x : $16K - 288 = -232 \Rightarrow K = 7/2$

(d) Comparing coefficients of x^2 : $B = 1944 - 288 \times 7/2 = 936$ [2]

6) (i) $-\pi < \theta \leq \pi$, $1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0$ [3]
 $\Leftrightarrow \cos\left(\theta - \frac{\pi}{5}\right) = \frac{1}{2}$
 $\Leftrightarrow \theta - \frac{\pi}{5} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{3} - 2\pi, \frac{5\pi}{3} - 2\pi$
 $\Rightarrow \theta = \frac{\pi}{5} + \frac{\pi}{3}$ and $\frac{\pi}{5} + \frac{5\pi}{3} - 2\pi$
 $\Rightarrow \theta = \frac{8\pi}{15}$ and $-\frac{2\pi}{15}$

(ii) $0 \leq x < 360^\circ$: $4\cos^2 x + 7\sin x - 2 = 0$ [6]

$\Leftrightarrow 4(1 - \sin^2 x) + 7\sin x - 2 = 0$
 $\Leftrightarrow 4 - 4\sin^2 x + 7\sin x - 2 = 0$
 $\Leftrightarrow 4\sin^2 x - 7\sin x - 2 = 0$
 $\Leftrightarrow (4\sin x + 1)(\sin x - 2) = 0$
 no solutions

$\sin x = -\frac{1}{4} \Rightarrow x = 180^\circ - \sin^{-1}(-1/4)$
 $x = 360^\circ + \sin^{-1}(-1/4)$

$\therefore x = 194.5^\circ, 345.5^\circ$

⑦ (a) Find $\int 3x - x^{3/2} dx$. [3]

$$\int 3x - x^{3/2} dx = \frac{3x^2}{2} - \frac{x^{5/2}}{(5/2)} + C$$

$$= \frac{3x^2}{2} - \frac{2x^{5/2}}{5} + C //$$

(b) Area $S = \int_0^9 3x - x^{3/2} dx = \left[\frac{3x^2}{2} - \frac{2x^{5/2}}{5} + C \right]_0^9$ [3]

$$= \frac{3 \cdot 81}{2} - \frac{486}{5} - 0 + 0$$

$$= \frac{243}{10} //$$

⑧ (i) $\log_3 (3b+1) - \log_3 (a-2) = -1$

$$\Leftrightarrow \log_3 \left(\frac{3b+1}{a-2} \right) = -1$$
 [3]
$$\Leftrightarrow \left(\frac{3b+1}{a-2} \right) = 3^{-1}$$

$$\Leftrightarrow \frac{3b+1}{a-2} = \frac{1}{3}$$

$$\Leftrightarrow 3(3b+1) = a-2$$

$$\Leftrightarrow 9b + 3 = a-2$$

$$\Leftrightarrow 9b = a-5$$

$$\Leftrightarrow b = (a-5)/9 //$$

(ii) $2^{2x+5} - 7(2^x) = 0$ [4]

Let $u = 2^x$

$$2^5 (2^x)^2 - 7(2^x) = 0$$

$$\Leftrightarrow 32u^2 - 7u = 0$$

$$\Rightarrow u = 0 \text{ or } u = \frac{7}{32}$$

$$\Rightarrow \underbrace{2^x = 0} \text{ or } 2^x = 7/32$$

no solutions $\Rightarrow x = \log_2 7/32$

$$= -2.19 \text{ (2 dp)}$$

⑨ (a) Angle $\angle EFA$ is $2\pi/3$ so area sector is $\frac{1}{2} \cdot x^2 \cdot \frac{2\pi}{3} = \frac{\pi x^2}{3} \text{ m}^2$. [2]

(b) $BC = y$ metres and area of enclosure is 1000 m^2 . [3]

Area equilateral triangle + Area rectangle + Area sector = 1000

so $\frac{x^2\sqrt{3}}{4} + 2xy + \frac{\pi x^2}{3} = 1000 \Leftrightarrow 2yx = 1000 - \frac{x^2\sqrt{3}}{4} - \frac{x^2\pi}{3}$

$$\Leftrightarrow y = \frac{1000}{2x} - \frac{x\sqrt{3}}{8} - \frac{x\pi}{6} = \frac{500}{x} - \frac{x}{24} (4\pi + 3\sqrt{3})$$

$$\textcircled{c} \quad p = 2x + 2y + x + 2\pi x/3 \quad [3]$$

$$= x \left(3 + \frac{2\pi}{3} \right) + \frac{1000}{x} - \frac{x}{12} (4\pi + 3\sqrt{3})$$

$$= \frac{1000}{x} - \frac{x}{12} (4\pi + 3\sqrt{3} - 36 - 8\pi)$$

$$= \frac{1000}{x} + \frac{x}{12} (4\pi + 36 - 3\sqrt{3})$$

$$\textcircled{d} \quad \frac{dp}{dx} = -\frac{1000}{x^2} + \frac{1}{12} (4\pi + 36 - 3\sqrt{3}) = 0 \quad [5]$$

$$\Leftrightarrow x^2 = \frac{12 \times 1000}{4\pi + 36 - 3\sqrt{3}} \Rightarrow x = 16.6339 \quad (\text{taking the positive root})$$

$$\therefore p_{\min} = 120.2 = \text{Rounded } 120 \text{ m (3 s.f.)}$$

$$\textcircled{e} \quad \frac{d^2p}{dx^2} = \frac{2000}{x^3} > 0 \quad \text{for all } x \in \mathbb{R}^+ \quad [2]$$